

State Shape Catalog

Every state must map onto itself under a turn or a mirror. A state has a capitol when each of its symmetries keeps exactly one square in place, and that square is the capitol. Below is every valid state from 1 to 7 squares, drawn once each: a shape's rotations and mirror images count as the same shape.

Capitols need an odd size

A mirror or half turn pairs up every square except the one it keeps, so 2, 4 and 6-square states never have one.

Lines must cross one square

A mirror line has to pass through exactly one square, and a turn has to center on a square.

54 valid shapes

21 have a capitol and 33 don't. Shapes with no symmetry, like the L-tetromino, aren't valid states.

 Capitol square

 State with no capitol

 Mirror line

 Half-turn center

 Quarter-turn center

1 square

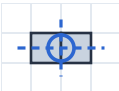
1 valid · 1 with a capitol
The single square is always its own capitol.



1 · 1 **Monomino**
● Capitol
2 straight mirrors · 2 diagonal mirrors · quarter turns
A single square is always its own capitol

2 squares

1 valid · 0 with a capitol
An even size, so no 2-square state can have a capitol.



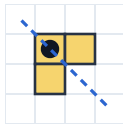
2 · 1 **Domino**
No capitol
2 straight mirrors · half turn
A mirror line crosses 2 squares

3

3 squares

2 valid · 1 with a capitol

An odd size, so a 3-square state can have a capitol.

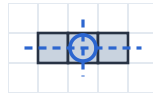


3·1 L-tromino

● Capitol

1 diagonal mirror

Capitol is the only square on the diagonal



3·2 Straight tromino

No capitol

2 straight mirrors · half turn

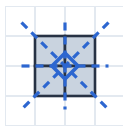
A mirror line crosses 3 squares

4

4 squares

4 valid · 0 with a capitol

An even size, so no 4-square state can have a capitol.

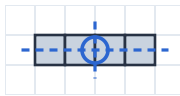


4·1 O-tetromino

No capitol

2 straight mirrors · 2 diagonal mirrors · quarter turns

A mirror line crosses 2 squares

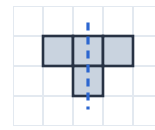


4·2 I-tetromino

No capitol

2 straight mirrors · half turn

A mirror line crosses 4 squares

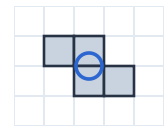


4·3 T-tetromino

No capitol

1 straight mirror

A mirror line crosses 2 squares



4·4 S-tetromino

No capitol

half turn

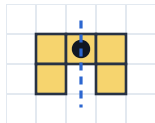
It turns around the middle of an edge, not a square

5

5 squares

7 valid · 4 with a capitol

An odd size, so a 5-square state can have a capitol.

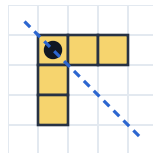


5·1 U-pentomino

● Capitol

1 straight mirror

Capitol is the only square on the mirror line

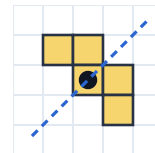


5·2 V-pentomino

● Capitol

1 diagonal mirror

Capitol is the only square on the diagonal

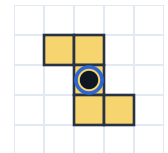


5·3 W-pentomino

● Capitol

1 diagonal mirror

Capitol is the only square on the diagonal



5·4 Z-pentomino

● Capitol

half turn

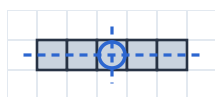
Capitol is the center of the half turn

6

6 squares

15 valid · 0 with a capitol

An even size, so no 6-square state can have a capitol.

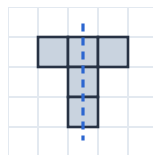


5·5 I-pentomino

No capitol

2 straight mirrors · half turn

A mirror line crosses 5 squares

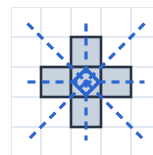


5·6 T-pentomino

No capitol

1 straight mirror

A mirror line crosses 3 squares

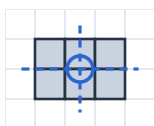


5·7 X-pentomino (plus)

No capitol

2 straight mirrors · 2 diagonal mirrors · quarter turns

A mirror line crosses 3 squares

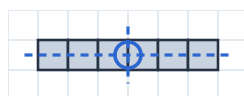


6·1

No capitol

2 straight mirrors · half turn

A mirror line crosses 2 squares

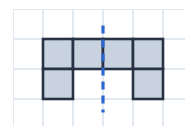


6·2

No capitol

2 straight mirrors · half turn

A mirror line crosses 6 squares

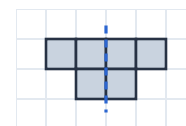


6·3

No capitol

1 straight mirror

A mirror line runs between squares

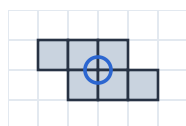


6·4

No capitol

1 straight mirror

A mirror line runs between squares

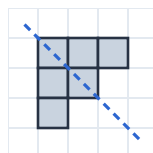


6·5

No capitol

half turn

It turns around a grid corner, not a square

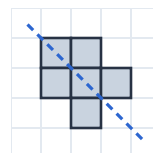


6·6

No capitol

1 diagonal mirror

A mirror line crosses 2 squares

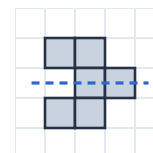


6·7

No capitol

1 diagonal mirror

A mirror line crosses 2 squares

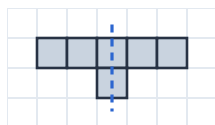


6·8

No capitol

1 straight mirror

A mirror line crosses 2 squares

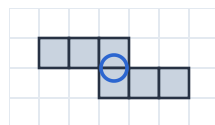


6·9

No capitol

1 straight mirror

A mirror line crosses 2 squares

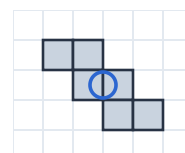


6·10

No capitol

half turn

It turns around the middle of an edge, not a square

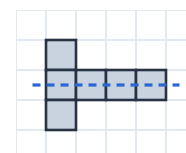


6·11

No capitol

half turn

It turns around the middle of an edge, not a square



6·12

No capitol

1 straight mirror

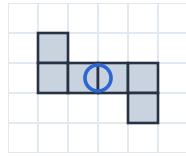
A mirror line crosses 4 squares

7

7 squares

24 valid · 15 with a capitol

An odd size, so a 7-square state can have a capitol.

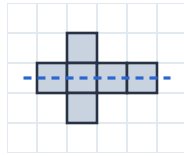


6 · 13

No capitol

half turn

It turns around the middle of an edge, not a square

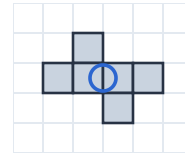


6 · 14

No capitol

1 straight mirror

A mirror line crosses 4 squares

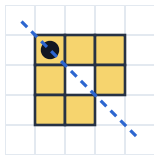


6 · 15

No capitol

half turn

It turns around the middle of an edge, not a square

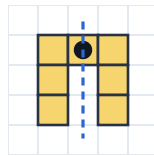


7 · 1

● **Capitol**

1 diagonal mirror

Capitol is the only square on the diagonal

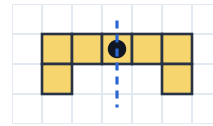


7 · 2

● **Capitol**

1 straight mirror

Capitol is the only square on the mirror line

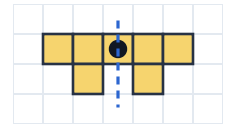


7 · 3

● **Capitol**

1 straight mirror

Capitol is the only square on the mirror line

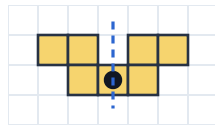


7 · 4

● **Capitol**

1 straight mirror

Capitol is the only square on the mirror line

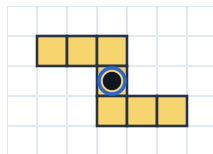


7 · 5

● **Capitol**

1 straight mirror

Capitol is the only square on the mirror line

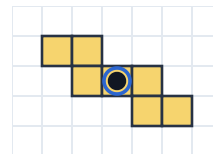


7 · 6

● **Capitol**

half turn

Capitol is the center of the half turn

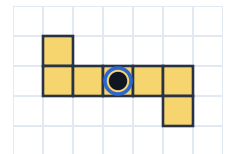


7 · 7

● **Capitol**

half turn

Capitol is the center of the half turn

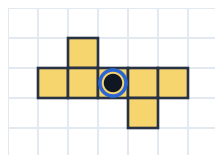


7 · 8

● **Capitol**

half turn

Capitol is the center of the half turn

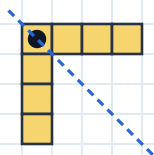


7 · 9

● **Capitol**

half turn

Capitol is the center of the half turn

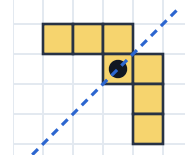


7 · 10

● **Capitol**

1 diagonal mirror

Capitol is the only square on the diagonal

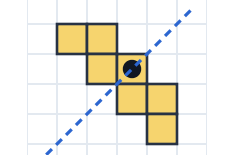


7 · 11

● **Capitol**

1 diagonal mirror

Capitol is the only square on the diagonal

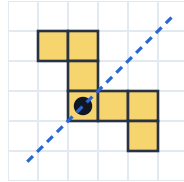


7 · 12

● **Capitol**

1 diagonal mirror

Capitol is the only square on the diagonal

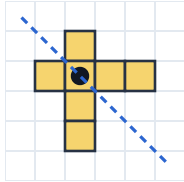


7 · 13

● Capitol

1 diagonal mirror

Capitol is the only square on the diagonal

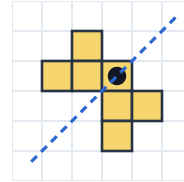


7 · 14

● Capitol

1 diagonal mirror

Capitol is the only square on the diagonal

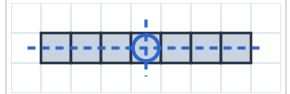


7 · 15

● Capitol

1 diagonal mirror

Capitol is the only square on the diagonal

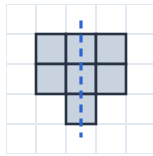


7 · 16

No capitol

2 straight mirrors · half turn

A mirror line crosses 7 squares

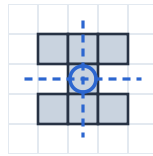


7 · 17

No capitol

1 straight mirror

A mirror line crosses 3 squares

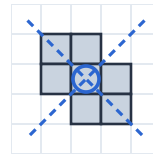


7 · 18

No capitol

2 straight mirrors · half turn

A mirror line crosses 3 squares

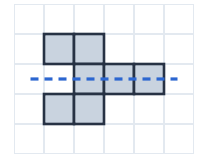


7 · 19

No capitol

2 diagonal mirrors · half turn

A mirror line crosses 3 squares

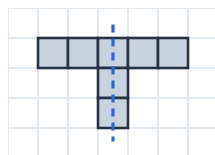


7 · 20

No capitol

1 straight mirror

A mirror line crosses 3 squares

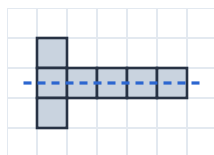


7 · 21

No capitol

1 straight mirror

A mirror line crosses 3 squares

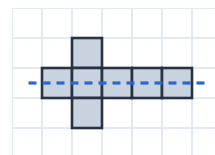


7 · 22

No capitol

1 straight mirror

A mirror line crosses 5 squares

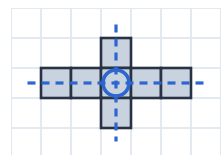


7 · 23

No capitol

1 straight mirror

A mirror line crosses 5 squares



7 · 24

No capitol

2 straight mirrors · half turn

A mirror line crosses 5 squares